Algebra Refresher and Using Units

In Science-1A, we learn about the scientific method, unique vocabulary used in science, examples of logical deduction, and how scientific knowledge can be summarized in simple formulas. Even though this course is relatively light in mathematics - no trigonometry, analytic geometry, or calculus – students have been required to have a course in algebra. Still, only a small amount of the algebra curriculum is really necessary, and that is primarily in the physics half of this course. In this note, I will review the methods of algebra that have challenged past students of Science-1A.

A central dot (·) is used in our notes to represent multiplication because the conventional symbol for multiplication, x, might become confused with the letter *x* that is often used to represent an unknown algebraic quantity. When we write scientific notation, however, we will use a distinct cross symbol, × between a number and the power of 10 as in 3.00×10^{8} .

The goal of this note is to end with the student being able to understand how to use Newton's Law of Gravitation and similar formulas. To that end a number of background steps are required:

1. Dividing fractions
2. Exponent Rules
3. Rearranging equations
4. Working with measurement units
5. Unit prefixes
6. Unit conversion
7. Scientific notation
8. Examples using scientific notation and units

When you have mastered these, you can work out any math problems in the physics part of this course.

1. Dividing Fractions

Example 1a: $\dfrac{2}{\frac{1}{2}}=2\cdot\dfrac{2}{1}=\dfrac{4}{1}=4$ **two pizzas with 1/2 for each person can server 4 people**

Example 1b: $\dfrac{\frac{3}{4}}{\frac{2}{5}}=\dfrac{3}{4}\cdot\dfrac{5}{2}=\dfrac{15}{8}$ **flip fraction in denominator and then multiply**

Example 1c: $\dfrac{\frac{5}{12}}{\frac{15}{16}}=\dfrac{5}{12}\cdot\dfrac{16}{15}=\dfrac{\cancel{5}}{3\cdot\cancel{2}\cdot\cancel{2}}\cdot\dfrac{\cancel{2}\cdot\cancel{2}\cdot2\cdot2}{3\cdot\cancel{5}}=\dfrac{4}{9}$ **factoring first and then cancelling**

Example 1d: $\dfrac{\frac{5}{12}}{\frac{15}{16}}=\dfrac{5}{12}\cdot\dfrac{16}{15}=\dfrac{80}{180}=\dfrac{4\cdot\cancel{20}}{9\cdot\cancel{20}}=\dfrac{4}{9}$ factoring last and then cancelling

2. Exponent Rules

When we calculate the area of a square with sides of length 5 meters (abbreviated as m), we multiply its width of 5 m times its height of 5 m and get and area of $A=(5\,\mathrm{m})\cdot(5\,\mathrm{m})=25\,\mathrm{m}^2$. For the volume of a cube of sides 5 m, we do $V=(5\,\mathrm{m})\cdot(5\,\mathrm{m})\cdot(5\,\mathrm{m})=125\,\mathrm{m}^3$. We write 25 as 5^2 and 125 as 5^3 and call these 5 squared and 5 cubed. The geometric interpretation of the exponents ^{2} and ^{3} of the 5 is clear, but the rules of mathematics actually allow exponents to be negative, zero, and fractional as well, without any clear geometic interpretation. We can see how this works with the following examples:

Example 2a: $2^2\cdot 2^3=(2\cdot 2)\cdot(2\cdot 2\cdot 2)=2\cdot 2\cdot 2\cdot 2\cdot 2=2^5=32$ We are just counting the number of 2's.

Example 2b: $(-3)^2\cdot(-3)^3=[(-3)\cdot(-3)]\cdot[(-3)\cdot(-3)\cdot(-3)]=9\cdot(-27)=-243=(-1)^5\cdot 3^5=-3^5$ Here, we are using the rules $(-1)\cdot(-1)=+1$ and $(-1)\cdot(+1)=(+1)\cdot(-1)=-1$. Odd powers of a negative are negative, and even powers are positive.

Example 2c: $x^2\cdot x^3=(x\cdot x)\cdot(x\cdot x\cdot x)=x\cdot x\cdot x\cdot x\cdot x=x^{(2+3)}=x^5$

We have found the rule that $x^2\cdot x^3=x^{(2+3)}=x^5$. Adding exponents performs a multiplication.

Example 2d: $\frac{2^5}{2^3}=\frac{2\cdot 2\cdot 2\cdot 2\cdot 2}{2\cdot 2\cdot 2}=\frac{2\cdot 2\cdot \cancel{2}\cdot \cancel{2}\cdot \cancel{2}}{\cancel{2}\cdot \cancel{2}\cdot \cancel{2}}=2\cdot 2=2^{(5-3)}=2^2=4$ The division subtracts from the final exponent.

Example 2e: $\frac{x^5}{x^3}=\frac{x\cdot x\cdot \cancel{x}\cdot \cancel{x}\cdot \cancel{x}}{\cancel{x}\cdot \cancel{x}\cdot \cancel{x}}=x^{(5-3)}=x\cdot x=x^2$ so we say $\frac{x^5}{x^3}=x^5\cdot x^{-3}=x^{[5+(-3)]}=x^{(5-3)}=x^2$

We have found the rule that $\frac{1}{x^3}=x^{-3}$. The reciprocal changes the sign of an exponent.

If we multiply both sides of this by x^3, we get $x^3\cdot\frac{1}{x^3}=x^3\cdot x^{-3}$ which is $1=x^{(3-3)}$ or $1=x^0$.

We have found the rule that $x^0=1$. Anything to the 0th power is 1, even $0^0=1$ (see Example 2n).

Example 2f: $\frac{x^3}{x^3}=\frac{\cancel{x}\cdot \cancel{x}\cdot \cancel{x}}{\cancel{x}\cdot \cancel{x}\cdot \cancel{x}}=1$ so we say $\frac{x^3}{x^3}=x^3\cdot x^{-3}=x^{[3+(-3)]}=x^{(3-3)}=x^0=1$. Same as previous result.

Example 2g: $\frac{x^4}{x^3}=\frac{x\cdot \cancel{x}\cdot \cancel{x}\cdot \cancel{x}}{\cancel{x}\cdot \cancel{x}\cdot \cancel{x}}=x$ so we say $\frac{x^4}{x^3}=x^4\cdot x^{-3}=x^{[4+(-3)]}=x^{(4-3)}=x^1=x$.

We have found the rule that $x^1=x$. Anything to the 1st power is itself.

Discovering Exponent Rules for Powers of Powers:

Example 2h: $(2^3)^5=(2\cdot 2\cdot 2)^5=8\cdot 8\cdot 8\cdot 8\cdot 8=32768=2^{15}$ We ended up with 5 groups of 3 totaling 15 2's.

Example 2i: $(x^3)^5=x^3\cdot x^3\cdot x^3\cdot x^3\cdot x^3=x^{(3+3+3+3+3)}=x^{(3\cdot 5)}=x^{15}$

We have found the rule that $(x^3)^5=x^{(3\cdot 5)}=x^{15}$. Powers of powers multiply.

Example 2j: $(x^3)^{-5}=\frac{1}{(x^3)^5}=\frac{1}{x^{15}}=x^{-15}$ or

$(x^{-3})^5=\frac{1}{x^3}\cdot\frac{1}{x^3}\cdot\frac{1}{x^3}\cdot\frac{1}{x^3}\cdot\frac{1}{x^3}=\frac{1}{x^3\cdot x^3\cdot x^3\cdot x^3\cdot x^3}=\frac{1}{x^{(3+3+3+3+3)}}=\frac{1}{x^{(3\cdot 5)}}=\frac{1}{x^{15}}=x^{-15}$ I**t works for negative powers as well.**

Example 2k: $(x^{-3})^{-5} = \frac{1}{(x^{-3})^5} = \frac{1}{x^{-15}} = x^{15}$ or simply $(x^{-3})^{-5} = x^{(-3)\cdot(-5)} = x^{+15} = x^{15}$

Even when both exponents are negative.

Example 2l: $\frac{\frac{y^2}{x}}{\frac{x^2}{y^3}} = \frac{y^2}{x} \cdot \frac{y^3}{x^2} = \frac{y^{(2+3)}}{x^{(1+2)}} = \frac{y^5}{x^3}$ Fliping and multiplying works, too (remembering $x = x^1$).

Example 2m: $(2^{\frac{1}{2}})^2 = 2^{\frac{1}{2} \cdot 2} = 2^1 = 2$ So when $2^{\frac{1}{2}}$ is squared, we get 2. So $2^{\frac{1}{2}}$ is the same as $\sqrt{2}$.

$\left(2^{\frac{1}{3}}\right)^3 = 2^{\frac{1}{3} \cdot 3} = 3^1 = 3$ So when $2^{\frac{1}{3}}$ is cubed, we get 3. So $2^{\frac{1}{3}}$ is the same as $\sqrt[3]{2}$.

So we see that fractional powers are generalizations of square root, cube root, etc.
This also works for decimal powers since a decimal can be written as a fraction:

$$\mathbf{0.52 = \frac{52}{100}} \quad \textbf{so} \quad \mathbf{2^{0.52} = \left(2^{\frac{1}{100}}\right)^{52} = (2^{52})^{\frac{1}{100}} = 1.4339552\ldots}$$ **(A calculator is required to get the answer.)**

The interesting case of 0^0:

Example 2n: What about 0^0? We will use a calculator with an x^y key that takes x to the y power and calculate the result of x^x as x gets closer and closer to 0 :

$$1^1 = 1.00000\ldots \quad , \quad 0.1^{0.1} = 0.79432\ldots \quad , \quad 0.01^{0.01} = 0.95499\ldots \quad , \quad 0.001^{0.001} = 0.99311\ldots \quad ,$$
$$0.0001^{0.0001} = 0.99907\ldots \quad , \quad 0.00001^{0.00001} = 0.99988\ldots \quad , \quad 0.000001^{0.000001} = 0.99998\ldots$$

From these, we may conclude that as we get closer to 0^0, the calculator's answers get closer to 1.00000...

More Examples:

Examples 2o: $2^1 = 2$ $\quad 2^2 = 2\cdot 2 = 4$ $\quad 2^3 = 2\cdot 2\cdot 2 = 8$ $\quad 2^4 = 2\cdot 2\cdot 2\cdot 2 = 16$

Examples 2p: $2^0 = 1$ $\quad 3^0 = 1$ $\quad 1.32^0 = \frac{132^0}{100^0} = \frac{1}{1} = 1$ $\quad (-1.62)^0 = (-1)^0 \cdot 1.62^0 = 1\cdot 1 = 1$ $\quad x^0 = 1$

Examples 2q: $2^{-1} = \frac{1}{2^1} = \frac{1}{2}$ $\quad 2^{-2} = \frac{1}{2^2} = \frac{1}{2\cdot 2} = \frac{1}{4}$ $\quad 2^{-3} = \frac{1}{2^3} = \frac{1}{2\cdot 2\cdot 2} = \frac{1}{8}$ $\quad 2^{-4} = \frac{1}{2^4} = \frac{1}{2\cdot 2\cdot 2\cdot 2} = \frac{1}{16}$

Example 2r: $\frac{x}{1/x} = \frac{x}{\frac{1}{x}} = x \cdot \frac{x}{1} = x^2$ or $\frac{x}{\frac{1}{x}} = \frac{x}{x^{-1}} = x \cdot x^1 = x^2$

Example 2s: $\frac{x^2}{\frac{1}{x^5}} = \frac{x^2}{x^{-5}} = x^2 \cdot x^5 = x^7$ or $\frac{x^2}{\frac{1}{x^5}} = x^2 \cdot \frac{x^5}{1} = x^2 \cdot x^5 = x^7$

Example 2t: $\frac{x^2}{x^5} = x^2 \cdot x^{-5} = x^{[2+(-5)]} = x^{(2-5)} = x^{-3}$

Example 2u: $\frac{x^3}{x^3} = x^3 \cdot x^{-3} = x^{[3+(-3)]} = x^{(3-3)} = x^0 = 1$ but $\frac{x^3}{x^{-3}} = x^3 \cdot x^3 = x^{(3+3)} = x^6$

3. Rearranging Equations

In algebra, we start with an equation that expresses the problem. Then we do things to that equation to get a useful answer. Whatever is done to left-hand side of the equal sign in the initial equation must also be done to the right-hand side with the end goal of an equation that states that the thing you are interested in (for example *x*) is at the left and everything else is suitably combined on the right side (or vise versa).

What you do at each step – add, subtract, multiply, divide, square, square root, reciprocal, etc. – is based on seeing what is necessary to get closer to your goal of having an equation that states *x*=... (or ...=*x*). Study the following examples to understand how this is done

Example 3a: Starting with the equation $2x+3=5$, find *x*.

Get rid of the 3 on the left by subtracting 3 from both sides: $2x+3-3=5-3$ to obtain $2x=2$.

Divide both sides by 2: $\frac{2x}{2}=\frac{2}{2}$ to obtain our answer $x=1$.

Just to be sure, check your answer by using *x*=1 in the original equation: $2\cdot 1+3=5$ and obtaining a true statement 5=5 . If you used the wrong value for *x* in this check, you would have obtained a false equality. For example, if you were to use $x=-1$, then the check would produce $2\cdot(-1)+3=5$ which produces the false statements $-2+3=5$ and finally $1=5$.

Example 3b: Starting with the equation $mgh=\frac{1}{2}mv^2$, find *v*.

Multiply both sides by 2 to get rid of the $\frac{1}{2}$ on the right side: $2\cdot mgh=2\cdot\frac{1}{2}mv^2$ to obtain $2mgh=mv^2$.

Divide both sides by *m* to get v^2 by itself on the right side: $\frac{2mgh}{m}=\frac{mv^2}{m}$ to obtain $2gh=v^2$.

Take the square root of both sides to get *v* by itself: $\sqrt{2gh}=\sqrt{v^2}$ to obtain $\sqrt{2gh}=v$.

Which is the same as saying $v=\sqrt{2gh}$.

Example 3c: Starting with the equation $f=\frac{1}{T}$, find *T*.

Take the reciprocal of both sides: $\frac{1}{f}=\frac{1}{\frac{1}{T}}$ to obtain $\frac{1}{f}=1\cdot\frac{T}{1}=T$.

Which is the same as saying $T=\frac{1}{f}$.

Example 3d: Starting with $c=\lambda f$, find λ. Here, λ is the Greek letter called lambda.

Divide both sides by f so that λ is by itself on the right side: $\frac{c}{f}=\frac{\lambda f}{f}$ to obtain $\frac{c}{f}=\lambda$.

Which is the same as saying $\lambda=\frac{c}{f}$.

Example 3e: Starting with $\frac{V_p}{N_p}=\frac{V_s}{N_s}$, find V_s .

Multiply both sides by N_s to obtain V_s by itself on the right: $N_s\cdot\frac{V_p}{N_p}=N_s\frac{V_s}{N_s}=\frac{N_s}{N_s}V_s=V_s$.

Which is the same as saying $V_s=\frac{N_s V_p}{N_p}$ or $V_s=\frac{N_s}{N_p}V_p$.

4. Measurement Units

Measured values need to have units, and science world-wide uses the metric system where:

The **basic unit** for **length** is the **meter**, abbreviated as m. The height of the diving board was 3 m.

The **basic unit** for **time** is the **second**, abbreviated as s. An extremely fast time for running 100 m is 10 s.

The **basic unit** for **mass** is the **kilogram**, abbreviated as kg. A typical person's mass is 70 kg.

Note: The mass unit gram g is 1000 times smaller than kg and is a unit of mass, but it is **not a basic unit**.

There are other units: **newton** (N) for force, **watt** (W) for power, **joule** (J) for energy, and many more. These additional units are called **derived** units since they are can be written in terms of a combination of the **basic** units. For example,

$$\mathrm{N}=\frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}^2} \qquad \mathrm{J}=\mathrm{N}\cdot\mathrm{m}=\frac{\mathrm{kg}\cdot\mathrm{m}^2}{\mathrm{s}^2} \qquad \mathrm{W}=\frac{\mathrm{J}}{\mathrm{s}}=\frac{\mathrm{kg}\cdot\mathrm{m}^2}{\mathrm{s}^3}$$

These derived units are named after scientists, but there is a peculiar international rule on capitalization of these names and abbreviations – the names are NOT to be capitalized, but the abbreviations MUST be capitalized.

Don't be concerned if you are unsure how to obtain the final numbers in the following examples. We will discuss that later in Section 7 entitled Scientific Notation. Just pay attention to how the units are handled.

Example 4a: When an object is moving at a constant velocity for a certain amount of time, we can use the formula $d=v\,t$ where *v* is the speed, *t* is the time, and *d* is the resulting distance travelled.

If the speed is 5.0 meters per second and the time is 2.0 seconds, we write $v=5.0\,\frac{\mathrm{m}}{\mathrm{s}}$ and $t=2.0\,\mathrm{s}$.

Using these values, we get

$$d=v\,t=(5.0\,\frac{\mathrm{m}}{\mathrm{s}})\cdot(2.0\,\mathrm{s})=5.0\cdot2.0\,\frac{\mathrm{m}}{\mathrm{s}}\cdot\mathrm{s}=10.0\,\mathrm{m}$$

Notice how the second of m/s in the speed was cancelled by the second unit of the time so that the only remaining unit was m for meters of distance.

Example 4b: If you run 1000 m in 500 seconds, what is your average speed. For this, the formula $d=v\,t$ can be used, but it must be rearranged using the rules of algebra that treat both sides of an equation equally. If both sides are divided by t, then the equation becomes

$$\frac{d}{t}=\frac{v\,t}{t}=v \quad . \text{ Reversing left and right sides, we have} \quad v=\frac{d}{t} \quad .$$

We can now put in the numbers with their units to obtain

$$v=\frac{1000\,\mathrm{m}}{500\,\mathrm{s}}=\frac{1000}{500}\,\frac{\mathrm{m}}{\mathrm{s}}=2\,\frac{\mathrm{m}}{\mathrm{s}} \quad \text{which can also be written as} \quad v=2\,\mathrm{m/s} \quad .$$

Example 4c: The density of an object is a measure of how tightly packed the mass of an object is. Wood is not very dense and floats in water, but rocks are more dense and sink. Density is mass represented by the letter *m* divided by a volume represented by the letter *V*. The traditional symbol for density is the Greek letter rho, ρ, which looks like a "p" but is curvy and written with a single sweep starting at the lower left. So, we have

$$\rho=\frac{m}{V} \quad .$$

To obtain the density of a block of aluminum with a mass of 20 kg and a volume of 0.007407 m^3, we use this new formula to obtain

$$\rho=\frac{m}{V}=\frac{20\,\mathrm{kg}}{0.007407\,\mathrm{m}^3}=2700\,\frac{\mathrm{kg}}{\mathrm{m}^3}=2700\,\mathrm{kg/m}^3 \quad .$$

Since the basic unit for length is the meter, the basic unit for volume is meter cubed, m^3. Using kg for mass and m^3 for volume gives us the basic units for density, kg/m^3.

Example 4d: If we need to carry water to a water trough at a remote part of a ranch and have a water tank with a volume of 3 m^3, we should check its mass to be sure that it will not overload our truck. Water has a density of close to 1000 kg/m^3, so to calculate the mass of this amount of water, we multiply both sides of the density formula by V to obtain the required formula:

$$V\cdot\rho = V\cdot\frac{m}{V} \quad \text{so} \quad V\cdot\rho = m \quad \text{, or reversing left and right,} \quad m=\rho V=(1000\,\frac{\text{kg}}{\text{m}^3})\cdot(3\,\text{m}^3)=3000\,\text{kg} \quad .$$

This will seriously overload pickup trucks! We may need to make 3 trips carrying 1 m^3 of water each time.

Example 4e: The force of gravitational attraction between the Earth and Moon is given by the formula discovered by Isaac Newton: $F=G\frac{m_{\text{earth}}\cdot m_{\text{moon}}}{r_{\text{earth-moon}}^2}$

where $G=6.67\times10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$ is a constant that is the same throughout the universe,

$m_{\text{earth}}=5.97\times10^{24}\,\text{kg}$ is the Earth mass,

$m_{\text{moon}}=7.35\times10^{22}\,\text{kg}$ is the Moon mass, and

$r_{\text{earth-moon}}=3.84\times10^{8}\,\text{m}$ is the distance between the center of the Earth and the center of the Moon.

Using these values in Newton's formula, we get

$$F=(6.67\times10^{-11}\frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{kg}^2}})\frac{(5.97\times10^{24}\,\cancel{\text{kg}})\cdot(7.35\times10^{22}\,\cancel{\text{kg}})}{(3.84\times10^{8}\,\cancel{\text{m}})^2}=1.985\times10^{20}\,\text{N}$$

Notice how nicely all units disappear except for N, the unit of force. Students often make the mistake of forgetting about the square in $(3.84\times10^{8}\,\text{m})^2$. Other students don't realize that the value of *G* with its units is provided on the Equation Sheet that they can used during Quizzes and Tests. One important lesson provided by studying science and math is how to pay close attention to all details! Making sure the units work out correctly helps check your work. Forgetting to square the distance in the denominator will make the final answer have units of N·m instead of just N. Leaving out G will cause the answer to have units of kg^2/m^2 instead of N.

Example 4f: A child on a swing might complete on full to-and-fro swing in 4 seconds. We say that 4 seconds is therefore the **period** time for a complete cycle. Only ¼ of a swing happens every second so we say the swing **frequency** is ¼ cycle per second, usually represented by a lower-case *f*. Instead of cycles per second, we now use the unit hertz (Hz) which is equivalent to reciprocal seconds, $\text{Hz}=\frac{1}{\text{s}}=\text{s}^{-1}$. The relation between frequency and period is the following very simple equation:

$$f=\frac{1}{T} \quad \text{or alternatively} \quad T=\frac{1}{f}$$

For our swing example, *T*=4 s and therefore $f=\frac{1}{4\text{s}}=\frac{1}{4}\text{s}^{-1}=\frac{1}{4}\text{Hz}$.

Or if we are told that $f=\frac{1}{4}\text{Hz}$, we can get the period as follows:

$$T=\frac{1}{f}=\frac{1}{\frac{1}{4}\text{Hz}}=\frac{1}{\frac{1}{4}\text{s}^{-1}}=\frac{4}{\text{s}^{-1}}=4\,s^{+1}=4\,\text{s} \quad .$$

Example 4g: Water waves also have a frequency. As you stand waist-deep in shark-free water with gentle, non-breaking waves flowing past you toward the shore, you feel the water rise and lower with a **frequency** f and a period T=1/f. But as you look from the shore at the waves, you will see that their crests and troughs move toward the shore at a certain velocity (speed) which we can represent by the letter v. The spacing between crests is a distance called the wavelength of the wave which we traditionally represent by the Greek letter lambda, λ. These three quantities can usually be connected by the following equation:

$$f=\frac{v}{\lambda} \quad \text{which might be rearranged to be} \quad v=f\,\lambda \quad \text{or} \quad \lambda=\frac{v}{f}$$

Water waves near the shore are actually rather complicated, so we prefer to talk about sound waves and electromagnetic (light and radio) waves. Sound waves typically have a velocity of v=340 m/s and electromagnetic waves move at the speed of light, 299792458 m/s, approximated by $3.00\times10^{8}\,\mathrm{m/s}$. However, instead of using the symbol v for the velocity of electromagnetic waves, tradition dictates the use of the lower case letter c.

Consider a light wave with a frequency of $5.0\times10^{10}\,\mathrm{Hz}$. We can calculate its wavelength as follows:

$$\lambda=\frac{c}{f}=\frac{3.00\times10^{8}\,\mathrm{m/s}}{5.0\times10^{10}\,\mathrm{Hz}}=0.60\times10^{-2}\frac{\mathrm{m/s}}{1/\mathrm{s}}=0.60\times10^{-2}\frac{\frac{\mathrm{m}}{\mathrm{s}}}{\frac{1}{\mathrm{s}}}=0.60\times10^{-2}\frac{\mathrm{m}}{\mathrm{s}}\cdot\frac{\mathrm{s}}{1}=0.60\times10^{-2}\,\mathrm{m}=0.0060\,\mathrm{m}$$

5. Unit prefixes

We have mentioned that the basic unit of mass is the kilogram (kg), not the gram which is 1000 times smaller. The kilo (k) in kilogram (kg) represents a factor of 1000. Consider the following equalities:

$$2\,\mathrm{kg} = 2 \cdot 1000\,\mathrm{g} = 2 \times 10^3\,\mathrm{g} \qquad 500\,\mathrm{kg} = 500 \times 10^3\,\mathrm{g} = 5 \times 10^2 \times 10^3\,\mathrm{g} = 5 \times 10^5\,\mathrm{g}$$

$$40\,\mathrm{km} = 40 \times 10^3\,\mathrm{m} \qquad 60000\,\mathrm{km} = 60000 \times 10^3\,\mathrm{m} = 6 \times 10^4 \times 10^3\,\mathrm{m} = 6 \times 10^7\,\mathrm{m}$$

Everywhere a "k" appears before a unit, it can be replaced by $\times 10^3$. Using the "k" prefix can make the number easier to read and remember.

Another common unit prefix is a lower-case m. One-thousandth of a meter is 0.001 m, but is is easier to think about it as 1 millimeter = 1 mm. Just as "kilo" or the prefix "k" means 1000, "milli" or the prefix "m" means 0.001.

A musical note commonly used when tuning musical instruments has a frequency of 440 Hz. Using the formula described earlier that $T = \frac{1}{f}$, we can calculate that the period between crests of voltage is $T = \frac{1}{440\,\mathrm{Hz}} = 0.002272727\ldots\,\mathrm{s} = 2.\overline{27}\,\mathrm{ms}$. Here the ms is to be read as milliseconds and interpreted as thousandths of a second.

All the prefixes you might need to know for this course or in understanding scientific and engineering literature are in the following table:

tera	T	$\times 10^{12}$	giga	G	$\times 10^{9}$	mega	M	$\times 10^{6}$	kilo	k	$\times 10^{3}$
centi	c	$\times 10^{-2}$	milli	m	$\times 10^{-3}$	micro	μ	$\times 10^{-6}$	nano	n	$\times 10^{-9}$
			pico	p	$\times 10^{-12}$	fempto	f	$\times 10^{-15}$			

The capitalization or not of these prefixes is important. T, G, and M are always capitalized; the others are never capitalized.

Here are some sentences using units with prefixes:

Cell phones now sometimes have processors with clock frequencies of 4 **G**Hz.
A desktop computer's main storage might hold 250 **G**Bytes of data, but a typical backup drive can hold 5 **T**Bytes. The fast memory storage capacity in a computer might be 16 **G**Bytes.
A Tesla coil can produce several **M**V of voltage and sparks that are several meters long.
Green light has a wavelength of 550 **n**m and a frequency of 545 **T**Hz.
An e-coli bacterium is a rod about 2 **μ**m long and 0.5 **μ**m in diameter.
A COVID-19 virus has a diameter of 120 **n**m.
The smallest object that we can see without a microscope or magnifying glass is about 100 **μ**m.
The smallest object visible in a microscope is limited by the wavelength of light to about 500 **n**m.
A hydrogen atom is about 100 **p**m in diameter, but its nucleus is only 1 **f**m in diameter.
A good camera has about 8 **M**pixels of resolution.
There are 100 **c**m in 1 m and 10 **m**m in each **c**m. So 1 m has 1000 **m**m.
Our brains process several **M**bits of data per second from our eyes, ears, and senses of touch and smell, but only tells our conscious awareness information at a rate of about 40 bits per second.
Light travels 30 **c**m in 1 **n**s.
Light travels 1 **k**m in $3.\overline{3}\,\boldsymbol{\mu}\mathrm{s}$.
Sound travels 1 **k**m in 2.9 s.
A bat doing echo location at a frequency of 50 **k**Hz is using sound with a wavelength of 6.9 **m**m.

6. Unit Conversion

If you plan to use an equation the requires kg, m, and s to get the correct answer and the numbers you have to start with have different units, you need to convert the values and units of the starting numbers. With practice, many conversions can be done effortlessly, but it is easy to get a conversion factor backwards. The method I describe here may seem involved, but can minimize the chance for error.

It is based on the idea the a 1 can be placed anywhere you wish and that 1 can be used to alter units.

Example 6a: You have a 3 g object and you need to know how much it weighs - that is how much force in newtons (See beginning of section 4: $\mathrm{N}=\frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}^2}$) is exerted on it by the gravitational force at the surface of the Earth.

The formula needed is $F=m g$ where $g=9.8\frac{\mathrm{m}}{\mathrm{s}^2}$.

Notice that there are two different uses of the letter g and two different uses of the letter m in this discussion. A "g" written without italics is the abbreviation for the unit "gram", but a "*g*" written in an italic font represents an acceleration. Similarly, a "m" written without italics is the abbreviation for the unit "meter", but a "*m*" written in an italic font represents a mass.

If you simply use $m=3\,\mathrm{g}$ in the formula $F=m g$, you get $F=(3\,\mathrm{g})\cdot(9.8\frac{\mathrm{m}}{\mathrm{s}^2})=29.4\frac{\mathrm{g}\cdot\mathrm{m}}{\mathrm{s}^2}$.

Unfortunately the answer needs to have units of newtons which are $\frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}^2}$.

You may say, "that's easy to fix, just use 0.003 kg for the mass." and you would be correct, but many students get it wrong and use 3000 kg instead. Here is how to not make that mistake if you remember your unit prefixes listed in Section 5.

You need kg, so recalling that the unit prefix k represents $\times 10^3$, write the equality $1\,\mathrm{kg}=1\times 10^3\,\mathrm{g}$.

This is an equation, so you can divide both sides by the same number and it will remain true. We will do this two different ways, but only one will be useful for this problem:

$$\frac{1\,\mathrm{kg}}{1\,\mathrm{kg}}=\frac{1\times 10^3\,\mathrm{g}}{1\,\mathrm{kg}} \quad \text{and} \quad \frac{1\,\mathrm{kg}}{1\times 10^3\,\mathrm{g}}=\frac{1\times 10^3\,\mathrm{g}}{1\times 10^3\,\mathrm{g}}$$

Anything divided by itself is 1 so these become

$$1=\frac{1\times 10^3\,\mathrm{g}}{1\,\mathrm{kg}} \quad \text{and} \quad \frac{1\,\mathrm{kg}}{1\times 10^3\,\mathrm{g}}=1$$

We now have two ways of representing 1, but the first way will convert kg to g, and the second will convert g to kg. We need to convert g to kg, so we use the second.

$$F=(3\,\mathrm{g})\cdot\mathbf{1}\cdot(9.8\frac{\mathrm{m}}{\mathrm{s}^2})=(3\,\mathrm{g})\cdot\left(\frac{\mathbf{1\,kg}}{\mathbf{1\times 10^3\,g}}\right)\cdot(9.8\frac{\mathrm{m}}{\mathrm{s}^2})=\frac{29.4}{10^3}\frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}^2}=29.4\times 10^{-3}\frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}^2}=29.4\times 10^{-3}\,\mathrm{N}$$

Example 6b: You have a formula for calculating the amount of heat in joules required to melt a mass of ice. Your ice has a mass of 5 kg. How many joules of heat are required?

If the formula to start with is $Q = m \cdot (334.88 \frac{\text{J}}{\text{g}})$, you will need to convert the mass from kg to g.

This time we need to use the other conversion factor found above:

$$Q = (5\,\text{kg}) \cdot \mathbf{1} \cdot 334.88\,\frac{\text{J}}{\text{g}} = (5\,\text{kg}) \cdot \left(\frac{\mathbf{1 \times 10^3\,g}}{\mathbf{1\,kg}} \right) \cdot 334.88\frac{\text{J}}{\text{g}} = \left(\frac{5 \times 1 \times 10^3 \cdot 334.88}{1} \right) \frac{\cancel{\text{kg}} \cdot \cancel{\text{g}} \cdot \text{J}}{\cancel{\text{kg}} \cdot \cancel{\text{g}}} = 1674.4 \times 10^3\,\text{J} = 1.6744 \times 10^6\,\text{J}$$

Here is a table with a few conversion factors:

Starting equality	**One Conversion**	**The other Conversion**
$1\,\text{mm} = 10^{-3}\,\text{m}$	$1 = \frac{10^{-3}\,\text{m}}{1\,\text{mm}}$	$1 = \frac{1\,\text{mm}}{10^{-3}\,\text{m}}$
$1\,\text{km} = 10^{3}\,\text{m}$	$1 = \frac{10^{3}\,\text{m}}{1\,\text{km}}$	$1 = \frac{1\,\text{km}}{10^{3}\,\text{m}}$
$1\,\mu\text{m} = 10^{-6}\,\text{m}$	$1 = \frac{10^{-6}\,\text{m}}{1\,\mu\text{m}}$	$1 = \frac{1\,\mu\text{m}}{10^{-6}\,\text{m}}$
$1\,\text{doz} = 12\,\text{units}$	$1 = \frac{12\,\text{units}}{1\,\text{doz}}$	$1 = \frac{1\,\text{doz}}{12\,\text{units}}$
$1\,\text{mol} = 6.022 \times 10^{23}\,\text{units}$	$1 = \frac{6.022 \times 10^{23}\,\text{units}}{1\,\text{mol}}$	$1 = \frac{1\,\text{mol}}{6.022 \times 10^{23}\,\text{units}}$
$1\,\text{cal} = 4.186\,\text{J}$	$1 = \frac{4.186\,\text{J}}{1\,\text{cal}}$	$1 = \frac{1\,\text{cal}}{4.186\,\text{J}}$
$1\,\text{food cal} = 1000\,\text{cal}$	$1 = \frac{1000\,\text{cal}}{1\,\text{food cal}}$	$1 = \frac{1\,\text{food cal}}{1000\,\text{cal}}$
$1\,\text{GHz} = 10^{9}\,\text{Hz}$	$1 = \frac{10^{9}\,\text{Hz}}{1\text{GHz}}$	$1 = \frac{1\text{GHz}}{10^{9}\,\text{Hz}}$

The starting equalities in the left column come from our unit prefix definitions (which you are expected to have memorized) or from page 2 of the Equation Sheet that you will have available during Quizzes and Tests.

That Equation Sheet is posted at
http://yosemitefoothills.com/Science-1A/EquationAndSymbolNotes.pdf

7. Scientific Notation

The enormous span of sizes and masses in the universe from atoms to intergalactic space requires that we often need to work with very large or small numbers. For that, we use scientific notation. Here is a table of examples to help you figure out for yourself a rule for converting between normal and scientific notation:

Normal	**Scientific**	**Normal**	**Scientific**	**Normal**	**Scientific**
1.2	1.2×10^{0}	0.12	1.2×10^{-1}	120000000	1.2×10^{8}
12	1.2×10^{1}	0.012	1.2×10^{-2}	0.000000000012	1.2×10^{-11}
120	1.2×10^{2}	0.0012	1.2×10^{-3}	12345	1.2345×10^{4}
1200	1.2×10^{3}	0.00012	1.2×10^{-4}	0.0000012345	1.2345×10^{-6}

And here are some more to check your rule:

Normal	**Scientific**	**Normal**	**Scientific**	**Normal**	**Scientific**
12960	1.296×10^{4}	0.101	1.01×10^{-1}	0	0.0×10^{0}
9.5	9.5×10^{0}	0.0032	3.2×10^{-3}	0.00030	3.0×10^{-4}
299792458	2.99792458×10^{8}	300000000	3.00×10^{8}	30000	3.0×10^{4}
0.0000000000667	6.67×10^{-11}	10	1.0×10^{1}	0.9	9.0×10^{-1}

These examples follow the tradition that the scientific notation will have the decimal point after the first non-zero digit, 6.67×10^{-11} rather than 0.667×10^{-10} or 66.7×10^{-12}. For the purposes of this course, however, any of these representations is perfectly correct. They all represent the same number in a compact form.

Your calculator will probably accept any of those and maybe even 0.0000000000667 as input values, but make it show your answers in scientific notation so that we do not have to count a ridiculous number of zeros. It is too easy to make errors counting zeros. Your calculator "number mode" should therefore be set to SCI (scientific). Different calculators will do that differently. Consult your calculator manual or get on-line help for you particular calculator model.

Also, different calculators have different ways of allowing entry of scientific notation numbers. For example, to enter 6.67×10^{-11}, many calculators would have you enter the keys 6 . 6 7 EE +/- 1 1 . Others might have a 10^x key for entering the -11 exponent. To access the EE or 10^x keys, a special "2nd" key might be required. Your cell phone probably has a calculator function which allows choices of style between "simple", "scientific", "financial", or "computer". You want the "scientific" style.

Here is a link to a YouTube video that discusses how 3 different calculators work when entering numbers in scientific notation: *https://www.youtube.com/watch?v=1jDfRhMl0z4*

Here are some examples of combining numbers with scientific notation. Notice that all parts of the number must be operated on. When squaring $3.00\times10^{8}\,\mathrm{m/s}$, you must do

$$\left(3.00\times10^{8}\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}=(3.00)^{2}\cdot(10^{8})^{2}\cdot\left(\frac{\mathrm{m}}{s}\right)^{2}=9.00\times10^{16}\frac{\mathrm{m}^{2}}{\mathrm{s}^{2}}$$

and when dividing, all parts divide

$$\frac{3.00\times10^{8}\dfrac{\mathrm{m}}{\mathrm{s}}}{5.0\times10^{-5}\mathrm{m}}=\frac{3.00}{5.0}\cdot\frac{10^{8}}{10^{-5}}\cdot\frac{\dfrac{\mathrm{m}}{\mathrm{s}}}{\mathrm{m}}=0.60\cdot10^{(8+5)}\cdot\frac{\cancel{\mathrm{m}}}{\cancel{\mathrm{m}}\,\mathrm{s}}=0.60\times10^{13}\frac{1}{\mathrm{s}}=6.0\times10^{12}\frac{1}{\mathrm{s}}=6.0\,\mathrm{THz}$$

Here, any of the last three versions of the answer is correct.

Numbers with units that are added or subtracted must have identical units. You cannot add apples and oranges. There is one peculiar exception to that rule. It has to do with the temperature units of celsius temperatures (ºC) and kelvin temperatures (K). These two temperature scales have exactly the same size units, but differ in their zero points so the equations $T_K = T_C + 273.15\,\mathrm{K}$ and $T_\mathrm{K} = T_C + 273.15\,^\circ\mathrm{C}$ give the same result. If $T_C = 22\,^\circ\mathrm{C}$, $T_K = 295.15\,\mathrm{K}$.

However, when cancelling temperature units, only K can cancel K and ºC can cancel ºC. For example, the power in watts radiated by a 5-m^2 black panel at 300 K is

$$P = (5.67\times10^{-8}\frac{\mathrm{W}}{\mathrm{m}^2\cdot\mathrm{K}^4})\cdot(5\,\mathrm{m}^2)\cdot(300\,\mathrm{K})^4 = 2.296\times10^3\,\mathrm{W} = 2296\,\mathrm{W}$$

The following expressions are clearly invalid because the units do not match:

$$d = 5\,\mathrm{m/s} + 6\,\mathrm{km/s} \quad \text{m and km do not match!}$$

$$v = v_0 + \frac{1}{2}a t^2 = 10\,\mathrm{m/s} + \frac{1}{2}(9.8\,\mathrm{m/s})\cdot(5\,s)^2 = 132.5\,\mathrm{m}/s \quad \text{mixed m/s and m}$$

$$f = \frac{1}{0.002\,\mathrm{s}} = 50\,\mathrm{kHz} \quad \text{mixed 1/s and kHz without conversion}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c}}} \quad \text{needs to be } \frac{v^2}{c^2} \text{ to combine with a dimensionless 1}$$

8. Examples Using Scientific Notation and Units

The question, formula used, and answer must be consistent!

If the questions asks "how fast", you must use a formula that starts with "v=" end up with an answer that has units of distance divided by time. (e.g. m/s)

If the question asks "how far", you must use a formula that starts with "d=" and end up with an answer that has units of distance. (e.g. m)

If the question asks "how much force", you must use a formula that starts with "F=" and end up; with an answer that has units of force. (e.g. N)

Example 8a: A car is initially moving at 10 m/s and then accelerates for 5 seconds at 4 m/s^2. How fast is it moving at the end of its acceleration?

$$v=v_0+at=10\,\mathrm{m/s}+(4\,\mathrm{m/s^2})\cdot(5\,\mathrm{s})=10\,\mathrm{m/s}+20\,\mathrm{m/s}=30\,\mathrm{m/s}$$

Example 8b: A rocket accelerates from launch at 10 m/s^2 for 50 s, how far has it gone when it stops accelerating?

$$d=\frac{1}{2}at^2=\frac{1}{2}\cdot(10\,\mathrm{m/s^2})\cdot(50\,\mathrm{s})^2=\frac{1}{2}\cdot(10\cdot 50^2)\cdot\frac{\mathrm{m}}{\mathrm{s}^2}s^2=1.25\times10^4\,\mathrm{m}$$

Example 8c: Einstein's famous equation $E=mc^2$ gives the energy that can be released when a mass m is converted completely to energy. Here c is the speed of light, $3.00\times10^8\,\mathrm{m/s}$. If we could convert a flea of mass 0.6 mg to pure energy, this formula gives the number of joules (J) of energy.

To use Einstein's formula, the flea mass must be converted to the basic mass unit, kg. This conversion is easily done wrong, so here we need to use the conversion factor $1=\frac{1\,\mathrm{kg}}{1000\,\mathrm{g}}$ to change g to kg:

$$m_{\mathrm{flea}}=0.6\,\mathrm{mg}=(0.6\times10^{-3}\,\mathrm{g})\times\left(\frac{1\,\mathrm{kg}}{1000\,\mathrm{g}}\right)=\frac{0.6\times10^{-3}\,\mathrm{kg}}{1000}=\frac{0.6\times10^{-3}}{10^3}\mathrm{kg}=0.6\times10^{-6}\,\mathrm{kg}$$

Now, we can finally get the energy:

$$E=mc^2=(0.6\times10^{-6}\,\mathrm{kg})\cdot(3.00\times10^8\,\mathrm{m/s})^2=(0.6)\cdot(3.00)^2\cdot10^{-6}\cdot(10^8)^2\frac{\mathrm{kg}\cdot\mathrm{m}^2}{\mathrm{s}^2}=5.4\times10^{10}\frac{\mathrm{kg}\cdot\mathrm{m}^2}{s^2}=5.4\times10^{10}\,\mathrm{J}$$

This is a lot of energy. A home might use an average of 2000 W over a year. Since a W is a J per second, we multiply the watts by the seconds in a year:

$$(2000\,\mathrm{J/s})\cdot(365.25\,\mathrm{day/year})\cdot(86400\,\mathrm{s/day})=6.31\times10^{10}\,\mathrm{J/year}$$

So the energy in the mass of the flea can power a home for nearly 1 year. We would, however, need another flea made of anti-matter to get that energy out, but then we would also get equal amount of energy from the mass of the anti-matter flea.