Areas and Volumes

A rectangle of length *l* and width *w* has an area $A_{\text{rectangle}} = lw$.

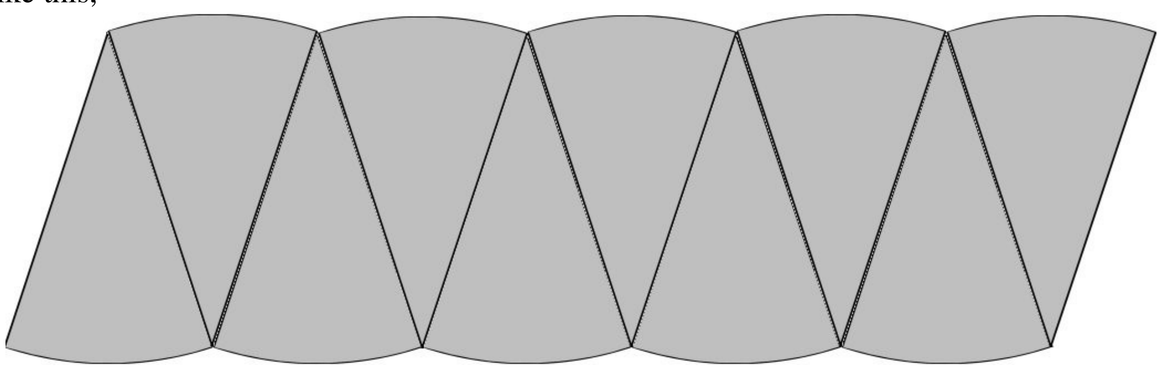
A square of side s is a special case of the rectangle with *l*=*w*=*s*, so its area is $A_{\text{square}} = s^2$.

A parallelogram (like a rectangle but adjacent sides are not perpendicular, kind of a squashed rectangle) of length *l* and height *h* is can be cut and pasted to form a rectangle of length *l* and height *h*. So the parallelogram has an area $A_{\text{parallelogram}} = lh$.

Any triangle with a base length *b* and a height *h* can be doubled to make a parallelogram with base *b* and height *h*. So a triangle has an area $A_{\text{triangle}} = \frac{1}{2} bh$, one-half of that parallelogram.

A trapezoid of height *h* and two parallel sides, *a* and *b*, where *b* is its shorter parallel side. It can seen as a triangle of height *h* and base *a-b* next to a parallelogram of height *h* and base *b*. The area is the sum of the area of the triangle $A_{\text{triangle}} = \frac{1}{2}(a-b)h$ and the parallelogram $A_{\text{parallelogram}} = bh$. This sum can be simplified, using the rules of algebra that we already know, to produce $A_{\text{trapezoid}} = \frac{1}{2}(a+b)h$ for the area of the trapezoid.

The **perimeter** of a circle (circumference) of radius *r* is $2\pi r$. From this and a little imagination, we can get the **area** of a circle. If, for example, we divide the circle into a 10 equally-sized sectors and arrange the sectors like this,



we nearly have created a parallelogram of height *r* and length πr , with half of the wedge-shaped sectors pointing up and the other half pointing down. The resulting parallelogram will be close to having an area of $A = \pi r^2$, which must also be the area of the circle since all we did was cut up and rearrange the circle.

Here are some volume formulas:

Cube: $V_{\text{cube}} = s^3$ Rectangular Solid: $V_{\text{rectangular solid}} = lwh$ Cylinder: $V_{\text{cylinder}} = \pi r^2 h$

Sphere: $V_{\text{sphere}} = \frac{4}{3}\pi r^3$ Cone: $V_{\text{cone}} = \frac{1}{3}\pi r^2 h$

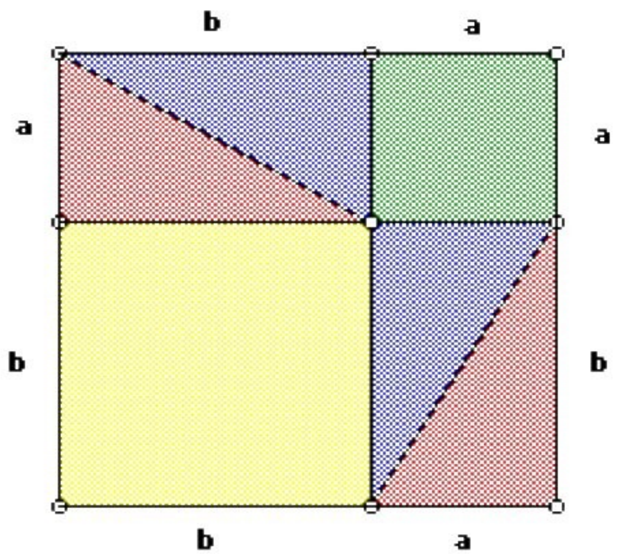
The formulas for a sphere and cone are obtained using calculus.

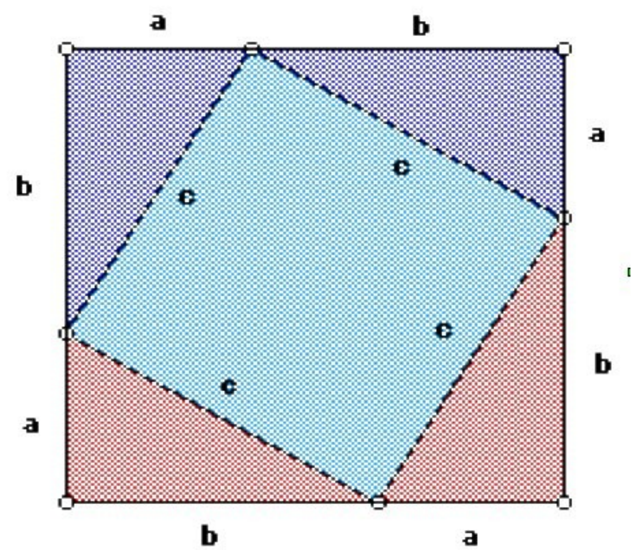
Special Note on the Pythagorean Theorem

The**e** Pythagorean Theorem applies to triangles of sides *a*, *b*, and *c* that have a 90° angle between sides *a* and *b*. It states that $a^2+b^2=c^2$. Here is the proof the Greek mathematician Pythagoras figured out 2500 years ago. The pictures came from:

http://jwilson.coe.uga.edu/EMT668/emt668.student.folders/HeadAngela/essay1/Pythagorean.html

where you can find more information about the Pythagorean theorem.





The area of the square on the left is given by $(a+b)^2$, or from its parts, $4(\frac{1}{2}ab)+a^2+b^2$.

The area of the square on the right is given by $(a+b)^2$, or from its parts, $4(\frac{1}{2}ab)+c^2$.

Since the squares have equal areas we can set these equal

$$4(\frac{1}{2}ab)+a^2+b^2=4(\frac{1}{2}ab)+c^2$$

Subtracting the $4(\frac{1}{2}ab)$ terms from both sides gives us the Pythagorean formula:

$$a^2+b^2=c^2$$

In problems, you might know two of the sides (*a* & *b*, *a* & *c*, or *b* & *c*) and need to find the remaining side (*c*, *b*, or *a*).