Analyzing Video Data of a Falling Rock

I made a 24-frame-per-second video of a rock dropping over 6 m distance, broke it into individual frames, and composed a single image so that we can test the book's equation for free-falling things on the surface of the earth. That image and a full image just as the rock was dropped are on the last 2 pages of these notes. The horizontal lines are separated by 1 meter and a black/white bulls-eye marks the bottom edge of the rock in each partial frame. One can measure the distance of the rock from the top line in each frame by studying this picture.

A real distance of 6.00 m corresponds to a distance of __137.3____mm on the paper.

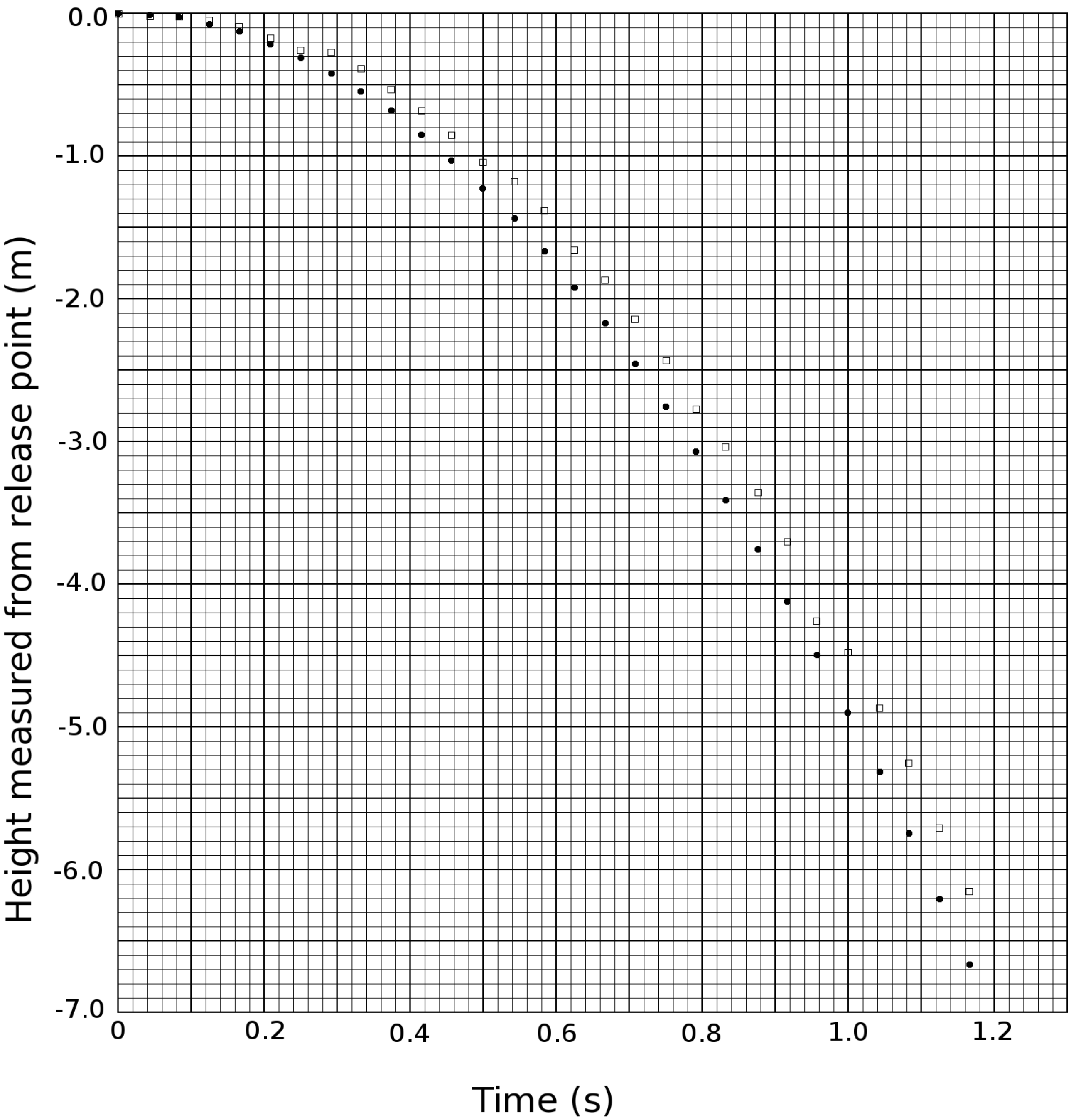
The scale factor for converting paper distance to real distance is then __0.0437__ m/mm.

To convert paper distance in mm to real distance in m, we multiply by this scale factor.

Use $g = 9.80\,\mathrm{m/s^2}$ when calculating the missing theoretical values for *h*.

Frame #	**Distance on Paper (mm)**	**Time (s)**	**Real Distance (m)**	$h = -\frac{1}{2} g t^2$	**Frame #**	**Distance on Paper (mm)**	**Time (s)**	**Real Distance (m)**	$h = -\frac{1}{2} g t^2$
0	0.00	0.000	0.000	-0.000	15	-38.0	0.625	-1.661	-1.914
1	-0.4	0.042	-0.017	-0.009	16	-42.8	0.667	-1.870	-2.178
2	-0.7	0.083	-0.031	-0.034	17	-49.0	0.708	-2.141	-2.459
3	-1.0	0.125	-0.044	-0.077	18	-55.72	0.750	-2.435	-2.756
4	-2.16	0.167	-0.094	-0.136	19	-63.52	0.792	-2.776	-3.071
5	-4.00	0.208	-0.175	-0.213	20	-69.47	0.833	-3.036	-3.403
6	-6.00	0.250	-0.262	-0.306	21	-76.9	0.875	-3.361	-3.752
7	-6.2	0.292	-0.271	-0.417	22	-84.7	0.917	-3.701	-4.117
8	-9.0	0.333	-0.393	-0.544	23	-97.4	0.958	-4.256	-4.500
9	-12.2	0.375	-0.533	-0.689	24	-102.58	1.000	-4.483	-4.900
10	-15.7	0.417	-0.686	-0.851	25	-111.5	1.042	-4.873	-5.317
11	-19.6	0.458	-0.857	-1.029	26	-120.37	1.083	-5.260	-5.751
12	-23.9	0.500	-1.044	-1.225	27	-130.50	1.125	-5.703	-6.202
13	-26.9	0.542	-1.176	-1.438	28	-140.91	1.167	-6.158	-6.669
14	-31.6	0.583	-1.381	-1.667					

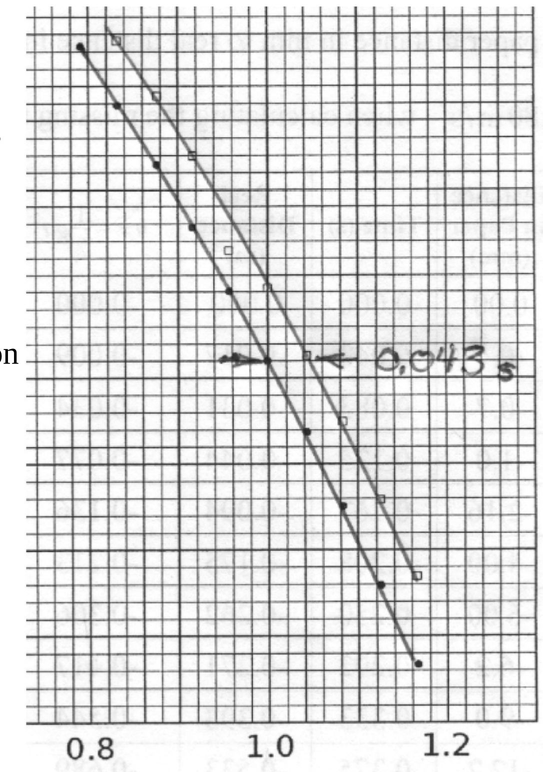
Now graph your results for the measured and calculated *h* vs *t* on the following graph vs. frame number and see how they compare:



By graphing our data and the formula together, it is clear that if we shifted our data to the left by about 1 frame (0.042 s), there would be excellent agreement between the two. It is very difficult to get the exact time of a hand release; an automatic dropping mechanism would greatly reduce that source of error.

Notice that the data points at 0.250 s and 0.958 s do not fit in smoothly with the others and should be re-examined.

If we draw smooth curves through the calculated points and measured points around the time of 1 s, we can estimate the time discrepancy between these curves to be 0.043 s. This is shown below

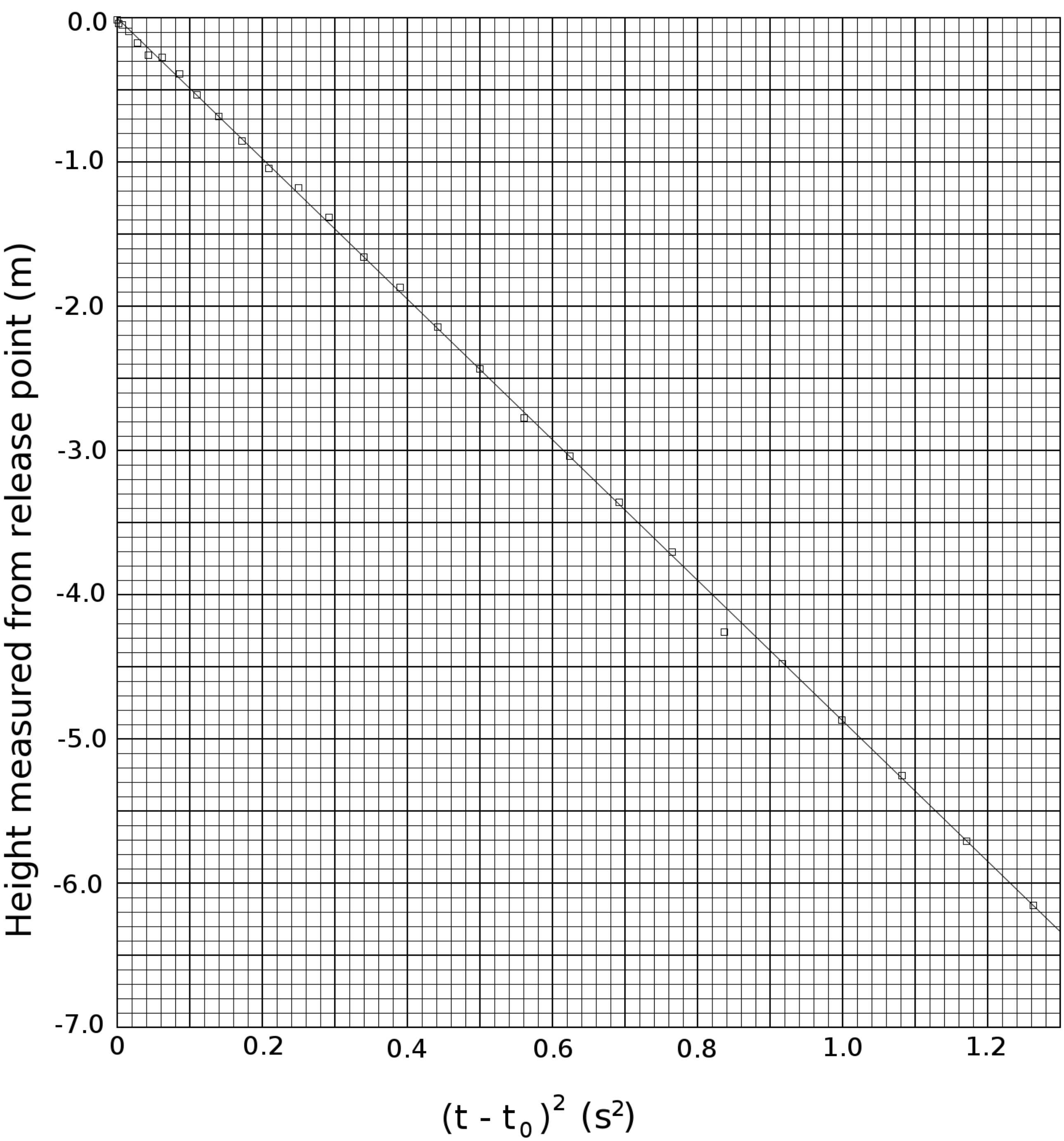


That is, the rock was actually released at a time t_0=0.043 s later than originally thought and we need to make a correction of that time error.

Since the theoretical behavior is that the distance fallen should vary as the square of the time since the rock was released, we will now make a correction for that error and then plot *h* versus the square of the corrected time:

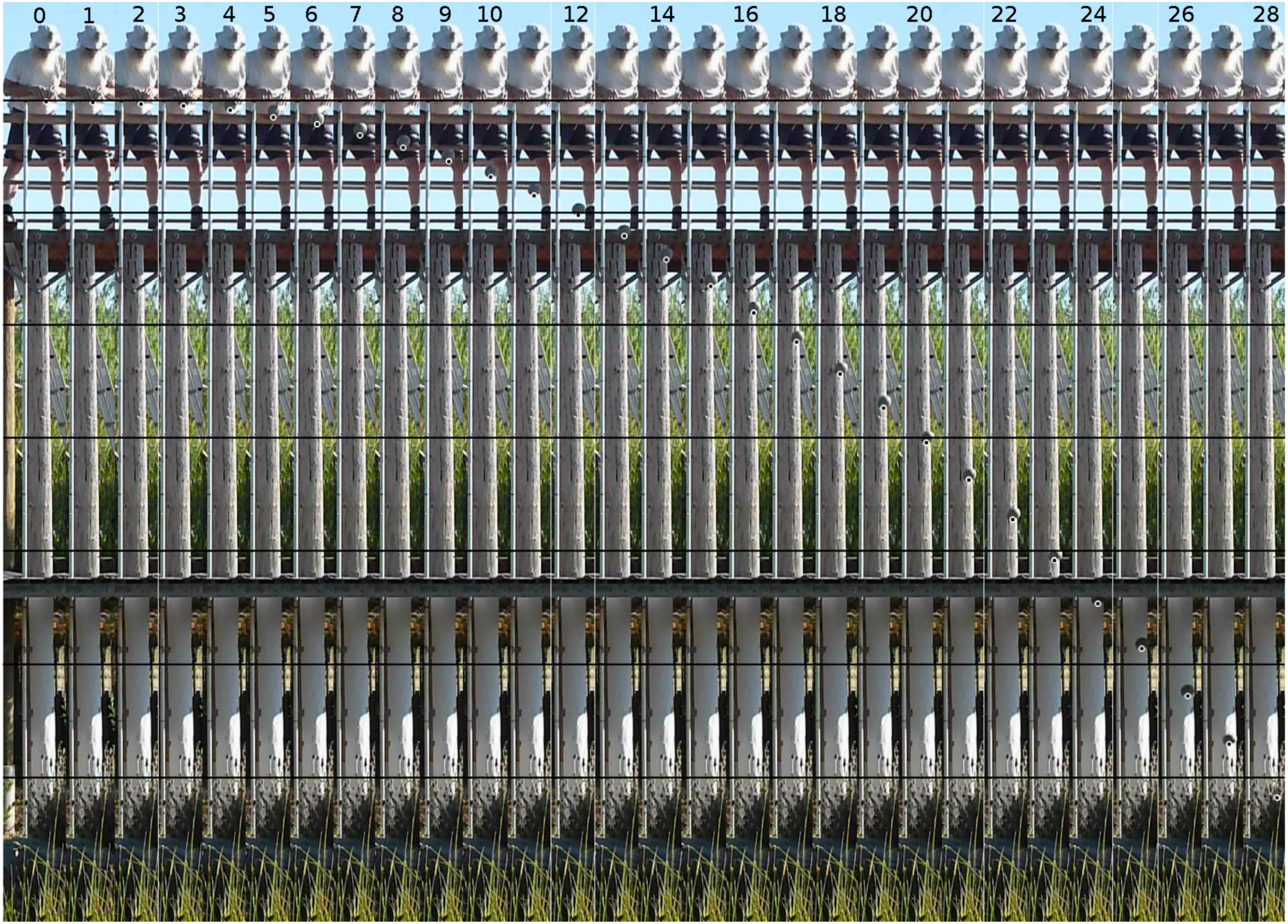
Frame #	**t (s)**	**$t - t_0$ (s)**	**$(t - t_0)^2$ (s)**	**h (m)**	**Frame #**	**t (s)**	**$t - t_0$ (s)**	**$(t - t_0)^2$ (s)**	**h (m)**
0	0.000	-0.043	0.002	-0.000	15	0.625	0.582	0.339	-1.661
1	0.042	-0.001	0.000	-0.017	16	0.667	0.624	0.389	-1.870
2	0.083	0.040	0.002	-0.031	17	0.708	0.665	0.442	-2.141
3	0.125	0.082	0.007	-0.044	18	0.750	0.707	0.500	-2.435
4	0.167	0.124	0.015	-0.094	19	0.792	0.749	0.561	-2.776
5	0.208	0.165	0.027	-0.175	20	0.833	0.790	0.624	-3.036
6	0.250	0.207	0.043	-0.262	21	0.875	0.832	0.692	-3.361
7	0.292	0.249	0.062	-0.271	22	0.917	0.874	0.764	-3.701
8	0.333	0.290	0.084	-0.393	23	0.958	0.915	0.837	-4.256
9	0.375	0.332	0.110	-0.533	24	1.000	0.957	0.916	-4.483
10	0.417	0.374	0.140	-0.686	25	1.042	0.999	0.998	-4.873
11	0.458	0.415	0.172	-0.857	26	1.083	1.040	1.082	-5.260
12	0.500	0.457	0.209	-1.044	27	1.125	1.082	1.171	-5.703
13	0.542	0.499	0.249	-1.176	28	1.167	1.124	1.263	-6.158
14	0.583	0.540	0.292	-1.381					

We now can plot our *h* data vs. the square of our corrected times as suggested by the theoretical formula:



The slope of this line is $\text{slope} = \frac{\Delta h}{\Delta(t-t_0)^2} = \frac{-6.325\,\text{m}}{1.300\,\text{s}^2} = -4.862\,\text{m/s}^2$. The theory says the data should follow the equation $h = -\frac{1}{2} g t^2$ so our slope is $-\frac{1}{2} g$ and therefore, our data gives

$$g = 2 \cdot 4.862\,\text{m/s}^2 = 9.724\,\text{m/s}^2$$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