Using Units

Base Units of Physics

mass usually represented by the letter ***m*** and having the base unit of **kilogram** (**kg**)
length usually represented by the letters ***d***, ***l*** or ***h*** and having the base unit of **meter** (**m**)
time represented by the letter ***t*** and having the base unit of **second** (**s**)

In the above definitions the letters representing the values are in italics whereas the units are not. This difference can be useful because some variables like speed ***s*** use the same letter as a base unit like time **s**.

Unit Multipliers

Units often involving multiplicative factors that are powers of 10. For example, 1000 meters can be written as 1000 m, but more simply can be written as 1 km where the k represents a multiplication by 1000. Here are more involving the length unit meter:

1 cm = 0.01 m = 10^{-2} m 1 mm = 0.001 m = 10^{-3} m 1 μm = 0.000001 m = 10^{-6} m 1 nm = 10^{-9} m

The set of prefixes is given below are the ones you will need to have **memorized** for this class:

tera T=10^{12} giga G=10^{9} mega M=10^{6} kilo k=10^{3}
centi c=10^{-2} milli m=10^{-3} micro μ=10^{-6} nano n=10^{-9} pico p=10^{-12} fempto f=10^{-15}

Notice that there is a Greek letter here called mu spoken as micro which has the shape μ, a bit like a backwards "y". It is written by starting with the lower left and ending at the upper right.

A frequent error is to think of the base unit of mass being the gram g, but it is not! The base unit of mass is actually the kg, 1000 times bigger. A microgram denoted by μg = 10^{-6} g = 10^{-9} kg.

Making metric unit conversions

Converting various lengths to meters. (The center dots and times X can be used interchangeable, but here I use both in a manner that I hope is not confusing.)

$$\mathbf{600\,km} = 600\cdot\mathbf{10^3}\,\mathrm{m} = 6\times10^2\cdot10^3\,\mathrm{m} = \mathbf{6\times10^5\,m}$$

$$\mathbf{0.006\,km} = 0.006\cdot\mathbf{10^3}\,\mathrm{m} = 6\times10^{-3}\cdot10^3\,\mathrm{m} = \mathbf{6\,m}$$

$$\mathbf{60\mu\,m} = 60\cdot\mathbf{10^{-6}}\,\mathrm{m} = 6\times10^1\cdot10^{-6}\,\mathrm{m} = \mathbf{6\times10^{-5}\,m}$$

$$\mathbf{0.06\mu\,m} = 0.06\cdot\mathbf{10^{-6}}\,\mathrm{m} = 6\times10^{-2}\cdot10^{-6}\,\mathrm{m} = \mathbf{6\times10^{-8}\,m}$$

Converting various lengths to nanometers - insert $\mathbf{1=10^9\cdot10^{-9}}$ in front of **m** after changing any other prefixes to powers of 10.

$$\mathbf{50\,m} = 5\times10^1\cdot\mathbf{1}\,\mathrm{m} = 5\times10^1\cdot(\mathbf{10^9\cdot10^{-9}})\,\mathrm{m} = 5\times10^1\times10^9\cdot(10^{-9}\,\mathrm{m}) = \mathbf{5\times10^{10}\,nm}$$

$$\mathbf{0.5\mu\,m} = 5\times10^{-1}\,\mu\mathrm{m} = 5\times10^{-1}\cdot\mathbf{10^{-6}}\,\mathrm{m} = 5\times10^{-7}\cdot\mathbf{1}\,\mathrm{m} = 5\times10^{-7}\cdot(\mathbf{10^9\cdot10^{-9}})\,\mathrm{m} = 5\times10^2\cdot(10^{-9}\,\mathrm{m}) = \mathbf{5\times10^2\,nm}$$

$$\mathbf{500\,km} = 5\times10^2\,\mathrm{km} = 5\times10^2\cdot\mathbf{10^3}\,\mathrm{m} = 5\times10^5\cdot\mathbf{1}\,\mathrm{m} = 5\times10^5\cdot(\mathbf{10^9\cdot10^{-9}})\,\mathrm{m} = 5\times10^{14}\cdot(10^{-9}\,\mathrm{m}) = \mathbf{5\times10^{14}\,nm}$$

$$\mathbf{500\,pm} = 5\times10^2\,\mathrm{pm} = 5\times10^2\cdot\mathbf{10^{-12}}\,\mathrm{m} = 5\times10^{-10}\cdot\mathbf{1}\,\mathrm{m} = 5\times10^{-10}(\mathbf{10^9\cdot10^{-9}})\,\mathrm{m} = 5\times10^{-1}(10^{-9}\,\mathrm{m}) = 5\times10^{-1}\,\mathbf{nm}$$

Using Units – Cartons of Eggs

When you buy 5 cartons, how many eggs do you have? Well, first you need to know how many eggs are in a carton. That might be written mathematically in any of the following ways

$$1 \text{ carton} = 12 \text{ eggs} \;\; \textbf{or} \;\; 12 \text{ eggs/carton} \;\; \textbf{or} \;\; \frac{1}{12}\,\text{carton/egg} \;\; \textbf{or} \;\; \frac{1\,\text{carton}}{12\,\text{eggs}} = 1 \;\; \textbf{or} \;\; \frac{12\,\text{eggs}}{1\,\text{carton}} = 1 \; .$$

If you buy 5 cartons, you might use the 2nd choice as follows: $(5\,\text{cartons}) \times (12\,\text{eggs/carton}) = 60\,\text{eggs}$.

This can also be written as $(5\,\text{cartons}) \times (12\,\frac{\text{eggs}}{\text{carton}}) = 60\,\text{eggs}$. The carton unit at the top cancels the one at the bottom just as the 4's cancel in $(3 \times \mathbf{4}) \times (\frac{5}{\mathbf{4}}) = 3 \times 5 = 15$.

Or you might use the 5th choice as follows: $(5\,\text{cartons}) \times \mathbf{1} = \left(\frac{5\,\text{cartons}}{1}\right) \times \left(\frac{\mathbf{12\,eggs}}{\mathbf{1\,carton}}\right) = 60\,\text{eggs}$.

If you want 48 eggs and need to know how many cartons to buy, you can use the 3rd choice and do

$$(48\,\text{eggs}) \times (\frac{\mathbf{1}}{\mathbf{12}}\,\mathbf{cartons/egg}) = \frac{48}{12}\,\text{cartons} = 4\,\text{cartons}$$

Or you might use the 4th choice and do $(48\,\text{eggs}) \times 1 = \left(\frac{48\,\text{eggs}}{1}\right) \times \left(\frac{\mathbf{1\,carton}}{\mathbf{12\,eggs}}\right) = \frac{48}{12}\,\text{cartons} = 4\,\text{cartons}$

Using Units – Liters and Cubic Centimeters of Liquid

When you buy 4 liters (L) of water, how many cubic centimeters (cm^3) of water do you have? Here, we need to know how many cubic centimeters are in 1 liter which can be written in the following ways:

$$1 \text{ L} = 1000 \text{ cm}^3 \;\; \textbf{or} \;\; 1000 \text{ cm}^3/\text{L} = 1 \;\; \textbf{or} \;\; \frac{1000\,\text{cm}^3}{1\,\text{L}} = 1 \;\; \textbf{or} \;\; \frac{1\,\text{L}}{1000\,\text{cm}^3} = 1 \;\; \textbf{or} \;\; \frac{1}{1000}\,\text{L/cm}^3 = 1$$

As with the egg-carton calculation, we can do

$$(4\,\text{L}) \times (1000\,\text{cm}^3/\text{L}) = 4000\,\text{cm}^3$$

or we can do

$$(4\,\text{L}) \times 1 = \left(\frac{4\,\text{L}}{1}\right) \times \left(\frac{1000\,\text{cm}^3}{1\,\text{L}}\right) = 4000\,\text{cm}^3$$

If we have 40 cm^3 and wish to know how many liters we have, we do

$$(40\,\text{cm}^3) \times \left(\frac{1}{1000}\,\text{L/cm}^3\right) = 0.040\,\text{L}$$

or we can do

$$(40\,\text{cm}^3) \times 1 = \left(\frac{40\,\text{cm}^3}{1}\right) \times \left(\frac{1\,\text{L}}{1000\,\text{cm}^3}\right) = 0.040\,\text{L}$$

Using Units – Converting km/h to m/s

Speed is expressed as distance per unit time. Sometimes it is given in km/h and a formula needs it in base units of m/s. To do the conversion, we need conversion factors for km to m and for hours to seconds. The prefix k in km immediately tells us one conversion factor that many be expressed in the following 4 ways:

$$1\,\text{km} = 1000\,\text{m} \quad \textbf{or} \quad \frac{1}{1000}\text{m} = 1\,\text{km} \quad \textbf{or} \quad \frac{1\,\text{km}}{1000\,\text{m}} = 1 \quad \textbf{or} \quad \frac{1000\,\text{m}}{1\,\text{km}} = 1$$

The other conversion factor deals with time – how many seconds in one hour. We know that there are 60 minutes in an hour and 60 seconds in a minute, so there are 60 x 60 = 3600 seconds in one hour. Using units makes this clearer:

$$60\,\text{min} = 1\,\text{h} \quad \textbf{or} \quad \frac{60\,\text{min}}{1\,\text{h}} = 1 \quad \textbf{or} \quad \frac{1\,\text{h}}{60\,\text{min}} = 1 \quad \textbf{and} \quad 60\,\text{s} = 1\,\text{min} \quad \textbf{or} \quad \frac{60\,\text{s}}{1\,\text{min}} = 1 \quad \textbf{or} \quad \frac{1\,\text{min}}{60\,\text{s}} = 1$$

so we can write

$$1 \times 1 = \frac{60\,\text{min}}{1\,\text{h}} \times \frac{60\,\text{s}}{1\,\text{min}} = \frac{60\,\text{min} \times 60\,\text{s}}{1\,\text{min} \times 1\,\text{h}} = \frac{60\,\cancel{\text{min}} \times 60\,\text{s}}{1\,\cancel{\text{min}} \times 1\,\text{h}} = \frac{3600\,\text{s}}{1\,\text{h}} = 1 = \frac{1\,\text{h}}{3600\,\text{h}}$$

Finally, to convert the speed of an earth satellite moving 25000 km/h to m/s, we do the following

$$\frac{25000\,\text{km}}{1\,\text{h}} = \frac{25000\,\text{km}}{1\,\text{h}} \times 1 \times 1 = \frac{25000\,\text{km}}{1\,\text{h}} \times \frac{1000\,\text{m}}{1\,\text{km}} \times \frac{1\,\text{h}}{3600\,\text{s}} = \frac{25000\,\cancel{\text{km}}}{1\,\cancel{\text{h}}} \times \frac{1000\,\text{m}}{1\,\cancel{\text{km}}} \times \frac{1\,\cancel{\text{h}}}{3600\,\text{s}} = 6.944 \times 10^3\,\text{m/s}$$

The form of the conversion factors was chosen in anticipation of the cancellations of km and h. Once you get familiar with this kind of manipulation, it will seem much simpler. You may simply remember other time conversion factors like $1\,\text{h} = 3600\,\text{s}$, $1\,\text{day} = 86400\,\text{s}$, and $1\,\text{year} = 356.24\,\text{days}$.

Units Help Us Catch Many Types of Formula Errors

Density is like a conversion factor between mass and volume for a particular material at a particular temperature and pressure. For example, the density of copper is $\rho = 8.96\,\text{g/cm}^3$ at normal temperatures and pressures. We have introduced another Greek letter called rho which is written as ρ. It looks similar to lower case p, and is written in a single stroke starting from the lower left and looping around to the middle left.

Treating the density of copper as a conversion factor, we can write

$$8.96\,\text{g} = 1\,\text{cm}^3 \quad \textbf{or} \quad 8.96\,\text{g/cm}^3 \quad \textbf{or} \quad \frac{1}{8.96}\text{cm}^3/\text{g} \quad \textbf{or} \quad \frac{1\,\text{cm}^3}{8.96\,\text{g}} = 1 \quad \textbf{or} \quad \frac{8.96\,\text{g}}{1\,\text{cm}^3} = 1$$

Aluminum, iron, wood, etc., all have different values of density so conversion between mass and volume are different for each type of material.

Other quantities in physics, like pressure and electrical resistance, sometimes appear to be simple conversion factors, but when studied carefully they are found to obey more complicate relationships.

The important thing to learn from this discussion is how units help you check your calculations. If you do a calculation of distance and your formula ends up giving you units that are not length, you have an error.

For example, a falling object obeys the formula $h = \frac{1}{2} g t^2$ where $g = 9.80\,\text{m/s}^2$ near sea level on the earth.

If the fall time is 3 s, the height is calculated using this formula as

$$h = \frac{1}{2} \cdot (9.80\,\text{m/s}^2) \cdot (3\,\text{s})^2 = \frac{1}{2}(9.80\,\frac{\text{m}}{\text{s}^2}) \cdot 3^2 \cdot \text{s}^2 = 44.10\,\text{m} \qquad \text{(correct)}$$

If, however, you were to forget the square of t, you would get

$$h = \frac{1}{2} \cdot (9.80\,\text{m/s}^2) \cdot 3\,\text{s} = \frac{1}{2}(9.80\,\frac{\text{m}}{\text{s}^2}) \cdot 3 \cdot \text{s} = 14.70\,\text{m/s} \qquad \text{(wrong!)}$$

m/s is a speed unit, not a distance which has units of m. That tells you that there is a mistake in your calculation.