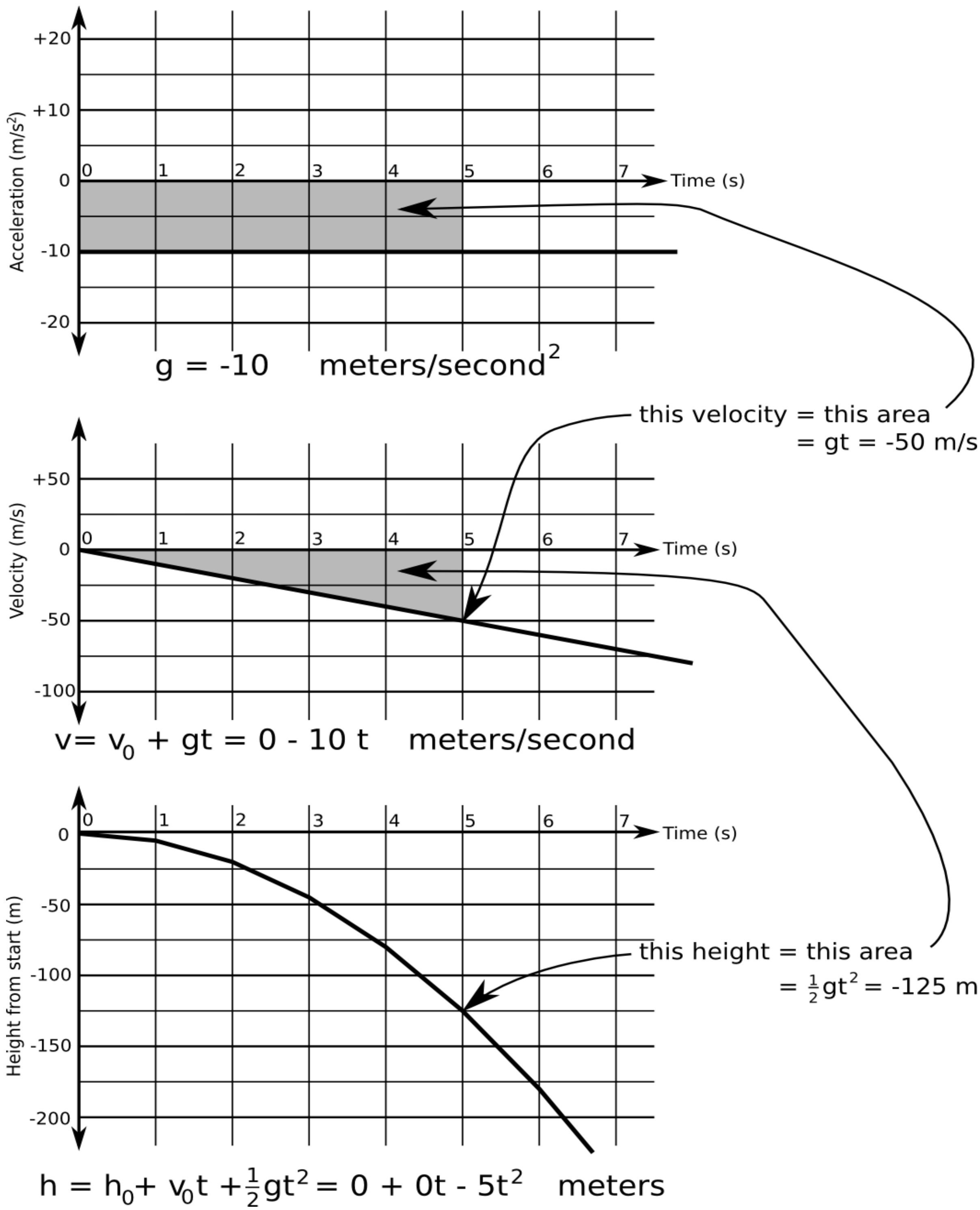
Acceleration, Velocity and Distance - Simple Case

The figures below show the acceleration, velocity, and vertical distance for an object dropped from near the surface of the earth using an acceleration of gravity of -10 m/s^2. Careful totaling of areas leads from the top graph, to the middle one, and then to the bottom graph. The area of each grid rectangle in the acceleration graph is 5 m/s while the area of each rectangle in the velocity graph is 25 m. The shaded area in the acceleration graph out to 5 s gives -50 m/s, the velocity at 5 s. The shaded area of the velocity graph out to 5 s gives -125 m, the drop in height at 5 s.



The Teacher Catching a Speeding Student

The student is moving down the hallway at a constant, unsafe speed of v_{student}. At time $t=0$, he passes a teacher. After a time *t*, he has moved beyond the teacher by a distance $d_{\text{student}}=v_{\text{student}}\,t$.

At time $t=0$, the teacher starts after the student with a constant acceleration *a*. After a time *t*, the teacher has moved a distance $d_{\text{teacher}}=\frac{1}{2}a_{\text{teacher}}\,t^2$. The ½ factor comes about because his speed is not constant, but increasing from 0 up to a speed of $v_{\text{teacher}}=a_{\text{teacher}}\,t$.

The teacher catches the student when $d_{\text{teacher}}=d_{\text{student}}$, or equivalently when $\frac{1}{2}a_{\text{teacher}}\,t^2=v_{\text{student}}\,t$. This last equation can be solved to get *t*, the time required to catch the student: $t=\frac{2\,v_{\text{student}}}{a_{\text{teacher}}}$. Using this time in either distance equation gives the distance required as

$$d_{\text{student}}=v_{\text{student}}\,t=\frac{2\,v_{\text{student}}^2}{a_{\text{teacher}}} \quad \text{or} \quad d_{\text{teacher}}=\frac{1}{2}a_{\text{teacher}}\,t^2=\frac{1}{2}a_{\text{teacher}}\left(\frac{2\,v_{\text{student}}}{a_{\text{teacher}}}\right)^2=\frac{2\,v_{\text{student}}^2}{a_{\text{teacher}}} \quad .$$

The velocities for the student and teacher are

$$v_{\text{student}} \quad \text{and} \quad v_{\text{teacher}}=a_{\text{teacher}}\,t=a_{\text{teacher}}\left(\frac{2\,v_{\text{student}}}{a_{\text{teacher}}}\right)=2\,v_{\text{student}}$$

Note: “velocity” means speed and direction, but in this problem all directions are the same and we did not need to worry about the distinction between velocity and speed.