Physical Concepts and Effects Related to Pendulums

Newton's 2nd Law: $F = ma$

Gravitational Force Law: $F_{\text{gravity}} = \frac{G\,m\,M_{\text{earth}}}{r_{\text{earth}}^2} = m\left(\frac{G\,M_{\text{earth}}}{r_{\text{earth}}^2}\right) = m\,g$ directed toward the center of the earth.

Here, $G = 6.67 \times 10^{-11}\,\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$ is the universal gravitational constant, $M_{\text{earth}} = 5.97219 \times 10^{24}\,\text{kg}$ is the mass of the earth, and $r_{\text{earth}} = 6371.0\,\text{km}$ is the mean radius of the earth. Together these make $g = 9.81\,\text{m/s}^2$.

Assuming no frictional loss, $E_{total} = E_{\text{kinetic}} + E_{\text{gravity}} = \frac{1}{2}m\,v^2 + m\,g\,\Delta h = \text{constant}$ where Δh is the height above the lowest point and *v* its speed at the lowest point.

The period of a pendulum *T* is related to its length *l* and the local acceleration of gravity *g* by the formula

$$T = 2\pi\sqrt{\frac{l}{g}}$$

A pendulum gradually dies down because of air resistance and friction in its support. This is called damped sinusoidal oscillation. For small swing angles $F_{\text{air}} \approx C\,v^2$ where *C* is some constant that depends on pendulum shape and air density. So if the pendulum swing is small, its average speed will be lower and the air resistance will be much less.

Imagine a pendulum that swings at very large angles, its motion will become very complex (nonlinear). It may reach a point where its supporting line becomes slack. If supported by a rod, it could even continue until it is inverted or swinging over the top of its support like a gymnast doing a hand giant.

A pendulum can swing to the left and right and also front and back or both at the same time leading to elliptical motion. If the two motions are one quarter of a period (90°) out of phase and of equal magnitude, the result is circular motion.

A pendulum with a fixed support swinging east and west, will attempt to continue its swing in the same plane, but as the earth rotates under it, it will appear to change direction. This is demonstrated with special pendulum called the Foucault pendulum which is given a small amount of energy to overcome friction. It can then swing long enough to see this effect.

Two pendulums weakly connected will transfer energy from one to the other except for two special modes of swinging. We will demonstrate this.

A ballistic pendulum can demonstrate momentum conservation. It catches a projectile and swings up to a height where it is caught by a ratchet mechanism.

Similar physics found in musical instruments, spring vibrations, vibrating atoms, and pogo sticks. The equations that govern their behavior are very similar.

A child pumping a swing and a singer breaking a crystal goblet are examples of putting energy into a pendulum in a regular way that reinforces its natural motion. That phenomenon is called resonance.

The flapping of a flag, the destruction of the Tacoma Narrows bridge, and the creation of a pure tone by rubbing a wet finger along the top of a crystal goblet are other examples creating pendulum-like motion.

Massive pendulums hidden within a skyscraper can help minimize the swinging of skyscrapers in the wind or during an earthquake.

Foucault Pendulum

A pendulum at the north or south geographic poles swings back and forth in a plane that does not rotate with respect to the stars. As the earth rotates under the pendulum, an observer imagines that the plane of the pendulum is rotating once in every 24 hours, but in reality the observer is rotating and the pendulum is not. When viewed from above the North Pole, the earth rotates counter-clockwise. As we ride on the earth's surface, we are moving from west to east; the sun is seen to set in the west. The plane of swing of a pendulum at the North Pole will appear to rotate in the opposite direction since we are accustomed to think that we are not moving when standing on the earth.

A pendulum at the equator does not appear to rotate. If it is swinging in the plane of the equator, its suspension point turns upside down as the earth rotates, but its plane of swing is unchanged. If it is swinging perpendicular to the equatorial plane, its suspension point drags it sideways causing it to trace very slight arcs, but does not cause it to appear to rotate.

At the latitude of Clovis, 36.8253° N, the plane of motion of a Foucault pendulum appears to rotate in a full circle every $\frac{24\,\text{hours}}{\sin 36.8253^\circ} = 40.042\,\text{hours}$. This corresponds to 360°/40.042 hours=8.991° per hour.

Unfortunately, the swing a pendulum gradually dies down as air resistance and friction at its support oppose its motion. These effects are reduced by using a smooth, spherical, and heavy mass (typically 100 kg) and a longer pendulum (typically 30 m). To completely counteract these effects and keep the pendulum swinging forever, some energy must be added to the pendulum just as a child must keep pumping a swing to keep it going. This “pumping,” however, must be done in a completely symmetrical manner so that it does not affect the angle of swing. Usually a carefully-constructed magnet is used.

Our pendulum is neither spherical, particularly heavy, nor very long, and has no pumping mechanism. We might just measure the loss of energy as it swings, and imagine how we might make it better.